Exercise 215

For the following exercises, evaluate the functions. Give the exact value.

$$\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$$

Solution

Take the tangent of $-\pi/6$.

$$\tan\left(-\frac{\pi}{6}\right) = \frac{\sin\left(-\frac{\pi}{6}\right)}{\cos\left(-\frac{\pi}{6}\right)} = \frac{-\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

So the aim is to find

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right).$$

The inverse tangent gives an angle between $-\pi/2$ and $\pi/2$.

$$x = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

$$\tan x = -\frac{1}{\sqrt{3}}$$

The value of x that satisfies this equation is $-\pi/6$. Therefore,

$$\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}.$$